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Distributed-Parameter Dynamics by Correlation Analysis

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The experimental dynamic response of a distributed-parameter, simultaneous heat and mass transfer system was investigated using correlation analysis. The system was a wetted-wall column operating as a nonadiabatic humidifier with both liquid and gas phases in turbulent flow.

The experimental frequency response results were compared to the values predicted by an existing mathematical model. The investigation also included a study of the effect of employing forcing functions having nonideal power spectra. Analog-simulated, first-order systems were tested to verify the computational procedure and to support the conclusions for the humidification system.

System dynamic identifications, acceptable for most engineering purposes, were obtained from forcing signals having power spectra which differed significantly from ideality. For the reduction technique employed, a time-domain record of approximately 2,500 pairs of input-output data points was found to suffice for a satisfactory analysis. The correlation technique yielded reliable results over approximately the same range of frequencies as reported for previous pulse test studies based on a comparable forcing procedure. However, the results showed an upper frequency limit below that previously achieved by direct frequency forcing of the same system.

The dynamic characteristics of chemical process systems are often obtained from differential equations derived from appropriate mass, energy, and momentum balances. However, if the process is too complex to permit mathematical modeling, or if verification of a model is desired, experimental testing techniques may be employed. Common experimental testing methods are direct frequency forcing, pulse testing, transient step response analysis, and correlation analysis.

The correlation technique employs a very complicated forcing function which may be considered to be the realization of random noise. In this sense, the term random is commonly used loosely to imply an irregular variation with time that cannot be predicted in advance.

The most attractive feature of correlation analysis is its validity for open-loop systems in the presence of corrupting noise which is uncorrelated with the system forcing function. Thus the technique is not limited, as are other experimental methods, by the necessity of maintaining all other process variables at steady values during the test. Another advantage is the possibility of finding a suitable disturbance already present in the input variable such that the process need not be artificially upset. Limitations of correlation analysis include the necessity of processing large amounts of data and the danger of steady state drifts yielding nonstationary functions which are unsuitable for system identification.

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A significant number of studies using correlation techniques for both real and simulated systems have been reported in the literature. Gallier, Slipcevich, and Puckett (*1*) and Wollaston and Swanson (*2*) have reported studies of essentially linear systems with single inputs. Goodman, Katz, Kramer, and Kuo (*3*) studied a stirred-tank blending operation in bench-scale hardware. Several investigators (*1, 4* to *6*) have tested processes with multiple inputs, whereas others (*3, 4*) have applied correlation techniques to nonlinear systems. An indication of the practical value of correlation analysis is given by the work of Hutchinson and Shelton (*7*), who identified an industrial distillation column operating in an in-plant environment. In spite of the fact that many industrial operations involve distributed-parameter processes, a large majority of the correlation analysis studies reported in the literature deal with lumped-parameter systems. Moreover, forcing functions having carefully controlled statistical properties have been almost universally employed even though it is logical to expect naturally occurring input disturbances to be something less than ideal.

In the present investigation, correlation techniques were used for the experimental thermal dynamic analysis of a wetted-wall column operating as a nonadiabatic humidifier. This air-water process is representative of a distributed-parameter system involving simultaneous heat and mass transfer. Previous studies (*8* to *13*) carried out on the same physical system and concerned with the formulation, solution, and verification of the system's mathematical models are extended. These previous studies employed both direct frequency forcing and pulse testing.

This investigation was initiated to accomplish three purposes: (1) to apply correlation analysis to a distributed-parameter system; (2) to compare the upper frequency limit of reliable results to that achieved by direct frequency forcing and pulse testing; (3) to employ forcing functions, having less than ideal properties, to obtain dynamic identifications. In an effort to produce conditions of a possible nonideal disturbance, a manually generated, arbitrary forcing function was used as the process input.

COMPUTATION PROCEDURE

The mathematical theory of correlation analysis has been previously reported in the literature (*1, 3, 4, 7, 14* to *17*). Therefore the computation procedure followed in this investigation is given without derivation.

The input and output time series, obtained by sampling a continuous record, were corrected by subtracting the mean from every value. The covariances were then computed as follows:

$$R_{fg}(\nu\Delta t) = \frac{1}{N+1-|\nu|} \sum_{k=0}^{N-|\nu|} f(k\Delta t)\ g(k\Delta t + |\nu|\,\Delta t) \tag{1}$$

R_{fg} is the autocovariance if f and g refer to the same series and the cross covariance if f and g refer to different series. The cross covariance was computed for the input both leading and lagging the output.

To reduce the truncation error resulting from the finite value of N in Equation (1), the covariances were multiplied by the Hanning lag window, defined as

$$D(\nu\Delta t) = 0.5\left(1 + \cos\frac{\pi\,|\nu|\,\Delta t}{T_m}\right) \tag{2}$$

The cross covariance between input and output was separated into even and odd parts as follows:

$$P_{xy}(\nu\Delta t) = 0.5\,[R_{xy}(\nu\Delta t) + R_{yx}(\nu\Delta t)] \tag{3}$$

and

$$Q_{xy}(\nu\Delta t) = 0.5\,[R_{xy}(\nu\Delta t) - R_{yx}(\nu\Delta t)] \tag{4}$$

The Fourier transform of a covariance is called the signal power spectrum and represents the link in correlation analysis between the time domain and the frequency domain. Employing the even symmetry of the autocovariance, we computed the input power spectrum as follows:

$$S_{xx}(\omega) = 2\int_0^{T_m} R_{xx}(\tau)\,\cos\omega\tau\, d\tau \tag{5}$$

The power spectrum, $S_{yy}(\omega)$ for the output, was computed in a similar manner. Again employing symmetry, we calculated the power spectrum for the cross covariance as

$$S_{xy}(\omega) = 2\int_0^{T_m} P_{xy}(\tau)\,\cos\omega\tau\, d\tau - 2i\int_0^{T_m} Q_{xy}(\tau)\,\sin\omega\tau\, d\tau \tag{6}$$

The harmonic integrals in Equations (5) and (6) were numerically evaluated using the Filon quadrature (*18*) which has been employed extensively for this type of integral.

Both real and imaginary parts of the system transfer function were then obtained as

$$G(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} \tag{7}$$

As a measure of the linear dependence between system input and output, the coherence was also computed as follows:

$$\gamma^2(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega)\,S_{yy}(\omega)} \tag{8}$$

A coherence of unity indicates no extraneous noise present in the output signal, whereas a coherence of zero indicates no correlation at all between input and output.

ANALOG SIMULATION STUDIES

To test the procedure for computation and data collection, an analog-simulated, first-order system having a time constant of 1 sec. was forced with a manually generated input signal. A continuous input-output record, 122.75 sec. in length, was obtained and sampled at 50-msec. intervals. Reduction of the data, using a maximum lag of 251 sample intervals, yielded the Bode diagrams presented in Figures 1 and 2.

In addition, time-domain data obtained by Halberg (*19*) for an analog-simulated, first-order system having a time constant of 2.5 sec. was reduced in a similar manner. A rectangular wave forcing function composed of pulses with identical amplitudes and a variable frequency of repetition was employed. This rectangular wave signal was obtained by modifying the output of a Gaussian noise generator. The continuous record, 170.1 sec. in length, was sampled at intervals of 100 msec. The results of this study are presented in Figures 3 and 4.

The agreement between experiment and theoretical prediction for both analog simulation studies is quite good over nearly three decades of frequency. In general, response data over three decades of frequency surrounding the breakpoint are considered adequate for a system identification.

EXPERIMENTAL INVESTIGATION

Equipment

The wetted section of the column used in this investigation was 1.5 ft. in length and had a 1-in. I.D. Stewart and Bruley (*9, 11*) give an equivalent description of the wetted-wall column and the air-water flow system used in the present work. A detailed cross section of the wetted-wall humidifier is shown in Figure 5.

The inlet and outlet air temperatures were sensed by copper-constantan thermocouples carefully matched to have equal time constants of 21 msec. in stagnant water. These thermocouple outputs were preamplified, passed through first-order filters, and recorded simultaneously on a highly sensitive, multiple-channel oscillograph. The first-order filters were used to eliminate preamplifier noise and had break frequencies of 66.6 rad./sec., well above the range of frequency interest.

Procedure

The system was allowed to reach steady state nonadiabatic conditions with both water and air streams in turbulent flow. The water stream falling film Reynolds number was 1,189, whereas the air stream Reynolds number was 2,520. Inlet air and water stream temperatures were 110° and 90°F., respectively. The inlet air temperature was then disturbed by feeding cooler air past a Nichrome heating filament located in the air stream. A switch in the heating filament circuit was manually operated to cause the recorded temperature traces to fluctuate nonuniformly about the steady state values. The maximum deviation of the inlet air temperature from steady state was limited to about 4.5°F., which previous pulse testing studies (*9* to *12*) have shown to be within the linear operating range of the wetted-wall humidifier.

Since the air stream thermocouples were not located exactly at the ends of the wetted section, it was necessary to test separately the inlet-outlet section as well as the total column arrangement. Since time-domain convolution transforms into frequency-domain multiplication, the total magnitude ratio for a linear system represents the product of the individual component magnitude ratios. Thus the magnitude ratio for the wetted section alone was obtained by dividing the magnitude ratio for the total column by the magnitude ratio for the inlet-outlet section. Five total column tests and two inlet-outlet section tests were conducted. A continuous record, approximately 130 sec. in length, was obtained from each test and sampled at 50-msec. intervals. The covariances for all runs were evaluated at lags between 0 and 251 sample intervals.

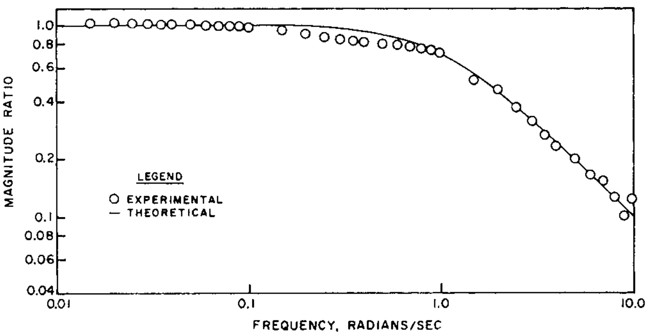

Fig. 1. Magnitude ratio for analog-simulated, first-order system (time constant = 1.0 sec.) from manually generated forcing function.

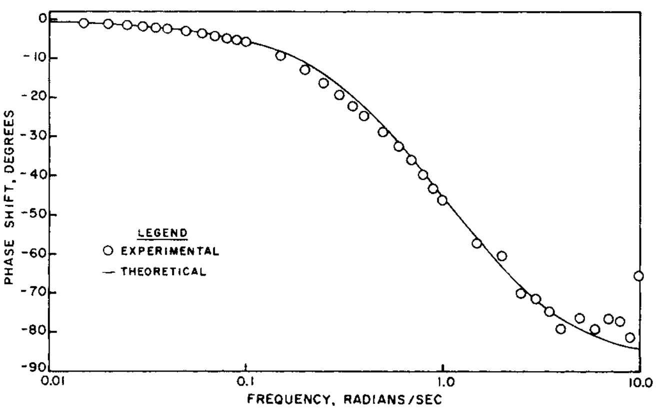

Fig. 2. Phase shift for analog-simulated, first-order system (time constant = 1.0 sec.) from manually generated forcing function.

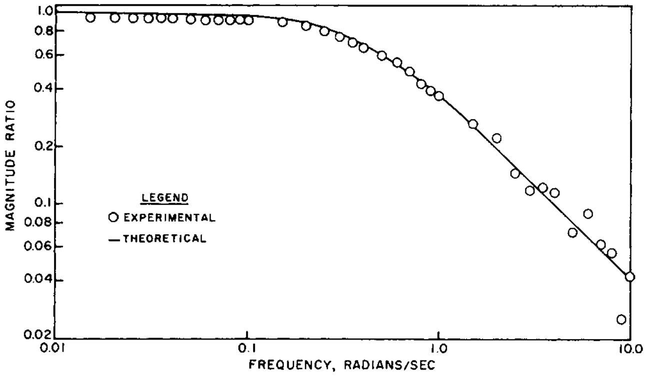

Fig. 3. Magnitude ratio for analog-simulated, first-order system (time constant = 2.5 sec.) from a rectangular wave forcing function.

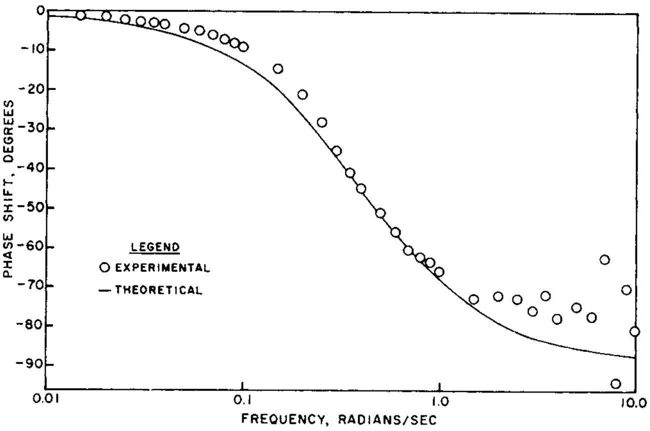

Fig. 4. Phase shift for analog-simulated, first-order system (time constant = 2.5 sec.) from a rectangular wave forcing function.

DISCUSSION OF RESULTS

Frequency Response Results

Magnitude ratio results for the nonadiabatic humidifier are presented in Figure 6. The total column values are arithmetic averages of four experimental runs, whereas the inlet-outlet section values are averages of two runs. The computed wetted section results may be compared directly to the theoretical response predicted from the mathematical model developed by Stewart and Bruley (*9*). Magnitude ratio values obtained from run 1 differed significantly from the other results and have been excluded from the averages. These values are presented separately in Figure 7 and will be discussed later. For most engineering purposes, the agreement between experimental and theoretical magnitude ratios is acceptable to a frequency of 1.5 rad./sec. Excluding run 1, individual values deviated from the reported average by less than 7%. In general, the accuracy of the results from correlation analysis compares quite favorably with previous pulse testing studies (*9* to *12*).

The constant magnitude ratio obtained experimentally is predicted by the point-forced, ideal plug-flow model. Since the ideal conditions of the model cannot be physically achieved, it is logical to expect a break point at some higher frequency. Unfortunately, the results of this investigation become unreliable before any break frequency is detected.

The experimentally determined phase shifts for the wetted-wall humidifier are presented in Figure 8 and are likewise in acceptable agreement with the mathematical model. Although the deviation from theoretical prediction is larger than for the corresponding magnitude ratios, the predominant characteristic of pure transport delay is clearly recognizable in the experimental response. The correlation technique failed to detect the small phase shift associated with the inlet-outlet section, and the values plotted are the uncorrected results for the total column. The failure to detect inlet-outlet section phase shift is not surprising since the anticipated correction was less than the experimental scatter.

The principal limitation of correlation analysis is the inherent error resulting from estimating covariances from finite records. This truncation effect is reduced by the use of a lag window but also requires that care be exercised in selecting the run length. The run lengths used in this investigation were based on criteria recommended by Homan and Tierney (4). Approximately 2,500 pairs of equally spaced input-output values were used to estimate the covariances. To verify that this choice of run length was adequate, the time-domain data from two total column runs were combined to form a record twice the normal length.

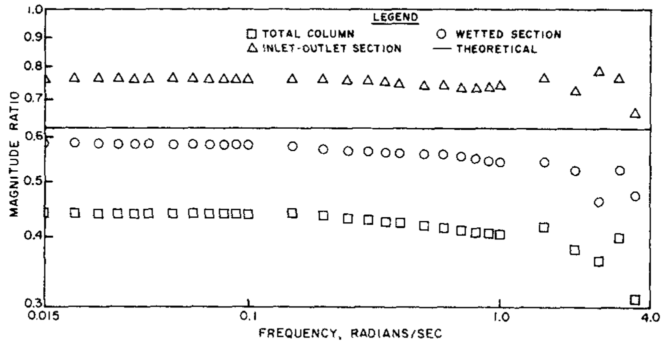

Fig. 6. Average magnitude ratio for turbulent nonadiabatic humidification at column centerline.

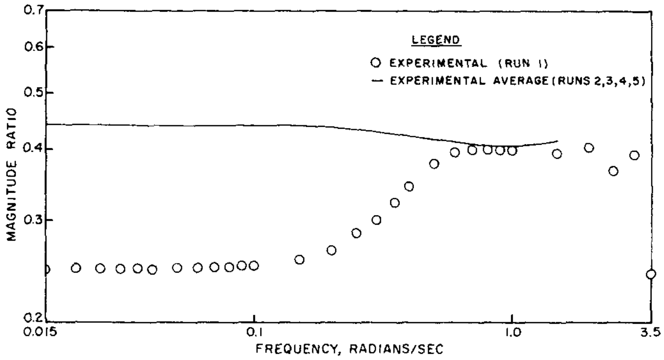

Fig. 7. Comparison of run 1 total column magnitude ratio with average of runs 2, 3, 4, and 5.

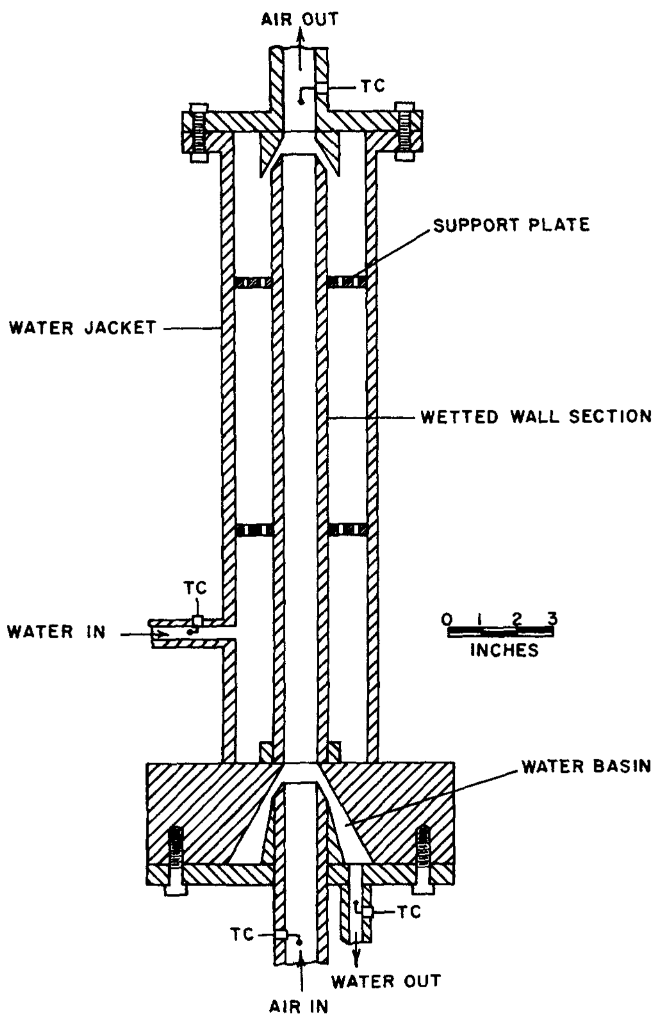

Fig. 5. Detailed cross section of wetted wall column.

The reduction of these data yielded values for both magnitude ratio and phase shift which did not deviate significantly from the results of the shorter runs. No attempt was made to determine whether shorter run lengths were possible.

Some interesting observations can be made concerning the magnitude ratio values obtained from total column run 1. The experimental results shown in Figure 7 deviate significantly from the other runs at low frequencies but agree quite well above 0.6 rad./sec. Surprisingly, the frequency range of discrepancy corresponds to the region of highest input power for run 1. In addition, it is instructive to consider the general trend observed in the coherences estimated from the experimental data. For all runs, except run 1, the coherence remained above 0.92 in the frequency range of significant input power. When the input power spectrum approached zero, the coherence diminished very rapidly. For run 1, however, the coherence was about 0.75 in the low frequency range, began a sharp increase at 0.3 rad./sec., and achieved values above 0.96 between 0.6 and 1.0 rad./sec. A direct relationship between the range of magnitude ratio agreement with the other four runs and the range of high coherences is apparent. It should be noted that, although not reported, the phase shift values from run 1 agreed closely with the other runs. This agreement would be expected since, as predicted by the mathematical model, the phase shift for the wetted-wall humidifier may be attributed to pure transport delay.

The results of run 1 are very similar to the results obtained by Goodman, Katz, Kramer, and Kuo (3) for a stirred-tank blending operation. They also obtained lower magnitude ratios than anticipated in the low frequency range and corresponding coherences as low as 0.8. Moreover, the experimental magnitude ratio curve for the blending tank showed the same approach to the theoretical curve as frequency increased. These investigators postulated that the discrepancy at the lower frequencies may have resulted

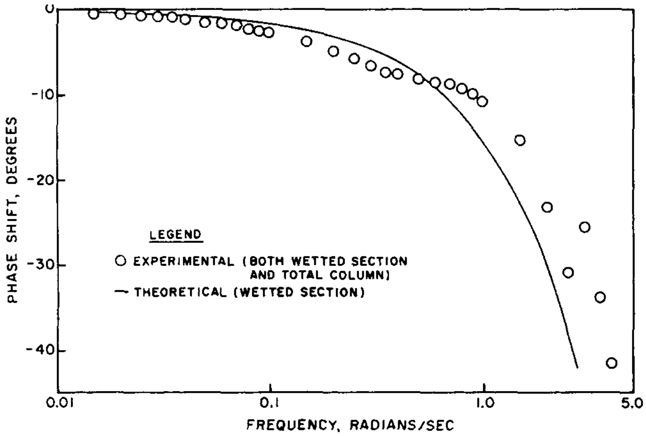

Fig. 8. Average phase shift for turbulent nonadibatic humidification at column centerline.

from slow drifts in the system operating conditions. Since such drifts would yield nonstationary functions, a similar explanation seems likely for run 1 of the present investigation. It is conceivable that because of the arbitrary nature of the forcing function a nonstationary signal may have been generated. Although a proven explanation of the run 1 discrepancy cannot be offered, the danger of basing an analysis on a single test is emphasized.

Forcing Function Properties

Although the experimental technique of manually generating forcing signals yielded no knowledge of the statistical distributions, the covariances and power spectra were computed from the recorded data and provide important descriptions of the functions actually employed. The covariances for all runs were remarkably similar in shape and in the value of the first zero. Typical plots, after multiplication by the Hanning lag window, are presented in Figure 9. Covariances are shown only for lags up to 150 sample intervals. For larger lags the covariances oscillated nearer and nearer to zero. Figures 10 through 12 present the input power spectra for all total column tests as well as the analog simulations.

The covariance and power spectrum plots may be used to justify the choice of analysis parameters. The covariance curves show that the choice of 251 lags for this investigation was adequate since the autocovariance has diminished to nearly zero at a lag of 150 sample intervals. The input power spectra curves indicate that the sample interval of 50 msec. was sufficiently small to eliminate any possibility of aliasing. The input power is essentially zero well before the corresponding Nyquist folding frequency of 62.8 rad./sec.

In general, an ideal input power spectrum has a constant value over the entire frequency range of interest. Such a spectrum can be obtained from band-limited white random noise. However, this experimental study yielded dynamic identifications in good agreement with theory from input power spectra differing significantly from ideality and varying in quality from one test to the next. This statement is strongly supported by a comparison of the input power spectra, shown in Figures 11 and 12, for the analog-simulated, first-order systems. An important observation from a comparison of these two analog simulation studies is that satisfactory identifications were obtained from forcing functions having significantly different power spectra. Of all the experimental runs, the power spectrum for the manually forced first-order system shows the largest deviation from ideality and yet produces the most precise frequency response results.

There is, however, some indication that spectral nonidealities may contribute to inaccuracies in experimental frequency response data. An incipient oscillation is present in the phase shift results from the wetted-wall column. Since an oscillation in this frequency range has not been previously detected by either direct frequency forcing or pulse testing, the occurrence for correlation analysis is probably related to some experimental inaccuray rather than to an inherent system characteristic. Oscillations would be expected for a distributed system forced in a distributed manner. However, for the pointwise forcing employed in this investigation, oscillations are not theoretically predicted. Noting that this oscillation occurs in the same frequency range as the spectral peak common to four of the experimental runs, a relationship seems possible. However, this explanation is inconclusive. All experimental runs for the total column, except run 3, yielded the characteristic phase shift oscillation, but the run 3 spectrum shows a more pronounced peak than does run 2. Moreover, the spectrum for the manually forced first-order system shows the most pronounced peak of all, and no distortion of the phase shift results was observed.

All input power spectra for the total wetted-wall column share a common characteristic of diminishing to zero at frequencies above 1.5 rad./sec. This approach to zero clearly corresponds to the point at which the experimental results begin to scatter on the Bode diagrams. This relationship is, of course, quite logical as no valid information can be anticipated at frequencies having negligible spectral power.

Experimental Frequency Limits

The persistent approach to zero of the input power spectra above 1.5 rad./sec., in spite of the arbitrary nature of the forcing signals, indicates that higher frequency components were prevented by physical limitations. Before entering the wetted section, the air stream passed through a calming section to allow the formation of uniform velocity profiles. The heat capacitance of this section limited the signal frequency content. Although rapid air stream temperature changes were possible at the inlet to the calming section, these fluctuations were attenuated severely before reaching the wetted section.

The results of this investigation show that correlation analysis yields reliable frequency response information to approximately the same upper frequency limit as pulse testing when the rate of change of the forcing variable is the same. Hunt (*12*) was unable to obtain reliable pulse test results for the wetted-wall humidifier above 1.5 rad./sec. This comparison is not surprising since identical forcing techniques were employed and the constraints of the

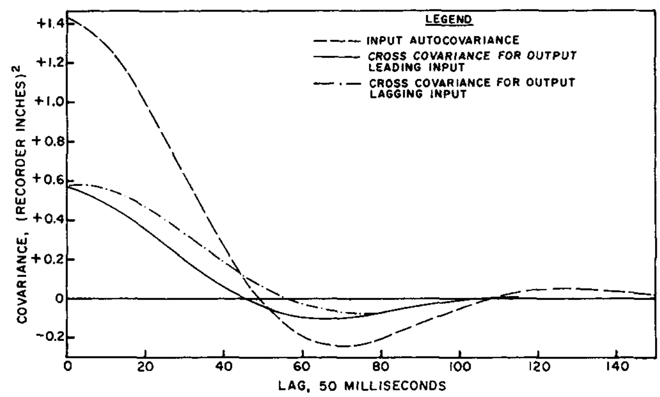

Fig. 9. Typical total column covariances.

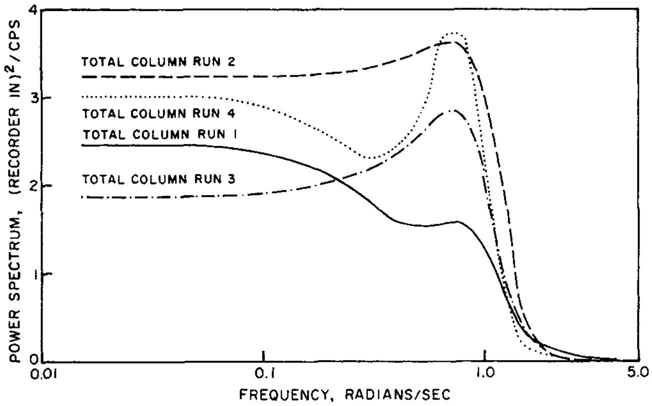

Fig. 10. Input power spectra obtained from manually generated forcing function.

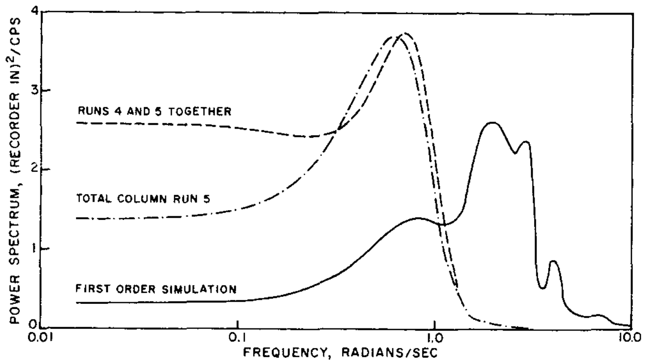

Fig. 11. Input power spectra obtained from manually generated forcing function.

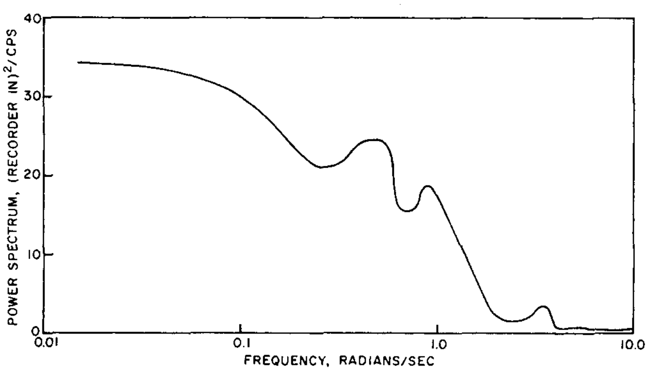

Fig. 12. Input power spectrum from rectangular wave forcing function.

calming section limited both investigations in the same manner.

Bruley (*13*) employed direct frequency forcing to obtain results for turbulent adiabatic humidification up to 2.24 rad./sec. Direct frequency forcing overcomes the problem of heat capacitance in the calming section by achieving a pseudosteady state. The frequency response results reported by Bruley were limited by the capabilities of the harmonic temperature generator.

Relative Merits of Experimental Testing Methods

Based on this study and previous investigations using the same system (*8* to *13*), a few comments are in order concerning the relative merits of various experimental testing techniques. For the wetted-wall humidifier, satisfactory agreement with theoretical predictions were obtained using direct frequency forcing, pulse testing, and correlation analysis.

Direct frequency forcing involved far less data handling and reduction effort than for the other two methods. Data reduction can actually be done by hand. However, considerably more experimental effort and equipment are required to generate a satisfactory sinusoidal forcing signal. All other process variables must be held at constant values during the test, and extraneous noise—both process and electrical—must be minimized. A practical limitation is the need to upset the process for relatively long periods of time.

Pulse testing offers the advantage of a simple forcing function which is easily generated and upsets the process only briefly. However, again noise must be minimized, and all other process variables must be held constant during the test. Although much less raw data need be handled than for correlation analysis, the technique is sufficiently lengthy to require a digital computer. For many applications where noise is not a problem and other process variables can be carefully controlled, pulse testing may well be the best compromise between experimental effort and data reduction complexities.

The principal advantage of correlation analysis is its ability to filter both random noise and random fluctuations in other variables. This advantage is of real value in an industrial environment. Moreover, a suitable forcing function may be naturally present in the input variable, eliminating the need to upset the process at all. If a suitable function is not present, a rather arbitrary one can be easily introduced. The main demerit of correlation analysis is the need to collect and reduce large amounts of raw data. However, for many complex systems with random disturbances present in a number of variables, correlation analysis may be the only feasible choice.

CONCLUSIONS

The application of correlation analysis to the experimental thermal dynamic study of a distributed-parameter, simultaneous heat and mass transfer system led to the following conclusions:

1. Correlation analysis is a satisfactory experimental technique for obtaining a dynamic identification of at least one distributed-parameter system. It is believed that correlation analysis can be applied successfully to many distributed-parameter systems in addition to the wetted-wall humidifier.

2. Satisfactory system identifications using correlation analysis may be obtained from input power spectra differing significantly from the ideal flat spectrum.

3. The results from correlation analysis are unreliable above the frequency at which the input power spectrum has diminished to nearly zero. This relationship is similar to that between normalized frequency content and reliable results from pulse testing.

4. For a wetted-wall humidifier, correlation analysis yields acceptable results over approximately the same range of frequencies as does pulse testing when the rate of change of the forcing variable is the same.

5. For a suitable sample interval and the computation procedure employed, a time-domain record of approximately 2,500 pairs of input-output data points yields a satisfactory analysis.

Further Information

For more detailed information, please refer to Ledbetter (*20*).

NOTATION

D = Hanning lag window
G = system transfer function
i = $+\sqrt{-1}$
k = integer
N = number of points sampled from continuous record
P_{xy} = even part of cross covariance, (in. of recorder deflection)^2
Q_{xy} = odd part of cross covariance, (in. of recorder deflection)^2
R_{fg} = covariance of functions f and g
R_{xx} = autocovariance of input signal, (in. or recorder deflection)^2
R_{xy} = cross covariance with input leading output, (in. of recorder deflection)^2
R_{yx} = cross covariance with input lagging output, (in. of recorder deflection)^2
S_{xx} = input power spectrum, (in. of recorder deflection)^2/(cycle) (sec.)
S_{yy} = output power spectrum, (in. of recorder deflection)^2/(cycle) (sec.)
S_{xy} = power spectrum of cross covariance, (in. of recorder deflection)^2/(cycle) (sec.)
T_m = maximum lag time, sec.

Greek Letters

γ^2 = coherence between input and output signals
Δt = sample interval, sec.
ν = integer assuming values between zero and maximum number of lags
τ = time lag, $|\nu|\Delta t$, sec.
ω = frequency, rad./sec.

Subscripts

f, g = general functions of time
x = system input signal
y = system output signal

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Activity Coefficients at Infinite Dilution for Ternary System

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A simple thermodynamic expression determining the infinite dilution ternary activity coefficients from the dew point isobar is presented. A flow method measuring the dew point temperature for ternary and binary isobaric systems is proposed. An analysis determining the infinite dilution ternary activity coefficients for ethanol-isopropyl alcohol-water system at 1 atm. is illustrated.

Knowledge of the infinite dilution ternary activity coefficients in the liquid phase is important for the design of distillation equipment, especially for the proper choice of solvent in distillation operation.

There have been several studies for experimentally determining the activity coefficients at infinite dilution. For binary systems the useful procedure, which is based on readily obtained data such as total pressure-concentration curve ($P - x$), bubble point-concentration curve ($T - x$) and dew point-concentration curve ($T - y$, or $P - y$), has been proposed by Carlson and Colburn (*5*), Redlich and Kister (*13*), Gautreaux and Coates (*7*), and Ellis and Jonah (*6*), and recently examined by Slocum and Dodge (*15*), and Kojima et al. (*9, 11*).

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